Lab 5 – Thermal Conductivity

PHYS 1020 – University of Manitoba

Written by: Scott Kehler

Abstract

The thermal conductivity of two good heat insulators is determined in this experiment, the insulators being rubber and cork. In order to conduct the experiment, an apparatus with two parts is used. The first part is an insulated copper cylinder, which acts as a source of heat. A thermocouple is attached to the copper cylinder to measure the change in temperature of the cylinder. The other part of the apparatus is a cold reservoir, which consists of a stainless steel container, with copper base, filled with water and ice. As heat flows from the relatively warm copper cylinder to the relatively cold ice-water container, the temperature of the copper cylinder drops. This drop in temperature is measured with time using a stopwatch and the thermocouple device. The experiment is conducted twice, once using a piece of rubber to insulate the copper cylinder from the ice-water container and the other time using cork to act as this insulator. In both cases, temperature changes are measured with time. The diameter of the copper cylinder and the thickness of the insulators are measured with a calliper in order to be used to calculate the thermal conductivity value. The value of thermal conductivity of rubber was calculated to be 0.33 ± 0.03 J / (m · s · C°) and the thermal conductivity of cork was calculated to be 0.061 ± 0.008 J / (m · s · C°).

Theory

Equations: Variables:

1) $\dfrac{\Delta Q}{\Delta t} = KA\dfrac{T_2 - T_1}{d}$

Q = Heat

K = Thermal Conductivity

2) $\Delta Q = -(\Delta Q)_{lost} = -MC_p\Delta T$

M = Mass

T = Temperature

3) $T_2 = T_0 \exp\left(-\dfrac{KAt}{dMC_p}\right)$

A = Cross-sectional Area

d = thickness

4) $\ell n T_2 = -\dfrac{KAt}{dMC_p} + \ell n T_0$

C_p =specific heat of copper

t = time

5) $K = -\dfrac{mdMC_p}{A}$

m = slope of best fit line

For this experiment, we are interested in calculating the value K, the thermal conductivity of a material. In order to do this we must come up with an equation for K, starting with the equation for heat conduction, equation 1:

$$\dfrac{\Delta Q}{\Delta t} = KA\dfrac{T_2 - T_1}{d}$$

We know that heat flowing through the insulator is equal to the negative of heat lost by the copper cylinder, which is equation 2:

$$\Delta Q = -(\Delta Q)_{lost} = -MC_p\Delta T$$

If we put equation 2 into equation 1, we get (using T_1=0°C, since that is the ice-water bath temperature):

$$\dfrac{-MC_p\Delta T}{\Delta t} = \dfrac{KAT_2}{d} \Rightarrow \dfrac{\Delta T}{T_2} = -\dfrac{KA\Delta t}{dMC_p}$$

Using calculus the above equation can be converted to the following (skipping the details):

$$T_2 = T_0 \exp\left(-\dfrac{KAt}{dMC_p}\right)$$

Taking the natural logarithm of both sides of the above equation, we get:

$$\ell n T_2 = \ell n \left(T_0 \exp \left(-\frac{KAt}{dMC_p} \right) \right)$$

This simplifies to:

$$\ell n T_2 = -\frac{KAt}{dMC_p} + \ell n T_0$$

Equation 4 can be considered a straight line equation. The equation of a straight line is y = mx + b. Based on that, consider the following:

y = $\ell n T_2$

m = $-\dfrac{KA}{dMC_p}$

x = t

b = $\ell n T_0$

Therefore, $-\dfrac{KA}{dMC_p}$ is equal to the slope of the line generated by this equation. As a result, we can calculate the value of the slope of the best fit line using a graph and determine the value of K by re-arranging the equation of m to the following:

$$K = -\frac{mdMC_p}{A}$$

Analysis – Calculations

(a) Calculate $\ell n T_2$

Take the natural logarithm of the observed temperature at time t:

      Ex) At time 0s: ln(20.0) = 3.00

Conduct this process for each temperature recorded

      *data from table 1, 2

(b) Calculate the slope of the best fit line for graph of rubber, lnT2 vs. time

Use equation: $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{1.95 - 2.77}{662 - 128} = -0.00154$$

      *data from graph 2 (Points P and Q)

(c) Calculate the slope of the worst fit line for graph of rubber, lnT2 vs. time

Use equation: $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{2.30 - 2.89}{425 - 65} = -0.00164$$

      *data from graph 2 (Points A and B)

(d) Calculate the slope of the best fit line for graph of cork, lnT2 vs. time

Use equation: $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{2.73 - 2.94}{2280 - 480} = -0.0001167$$

      *data from graph 4 (Points P and Q)

(e) Calculate the slope of the worst fit line for graph of cork, lnT2 vs. time

Use equation: $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{2.85 - 2.96}{1200 - 360} = -0.000131$

*data from graph 4(Points A and B)

(f) Calculate Δm for rubber

Use equation: $\Delta m = \left| m_{best} - m_{worst} \right|$

Δm = |-0.00154 – (-0.00164)| = 0.0001

*data from parts (b), (c)

(g) Calculate Δm for cork

Use equation: $\Delta m = \left| m_{best} - m_{worst} \right|$

Δm = |-0.0001167 – (-0.000131)| = 0.0000143

*data from parts (d), (e)

(h) Convert specific heat of Copper

Convert 0.093 calories per gram per degree C to joules per kilogram per degree C

$\dfrac{0.093 cal}{1g} = \dfrac{1000g}{1kg} \cdot \dfrac{4.19J}{1cal} = \dfrac{389.67J}{1kg}$

Specific heat of copper in SI units: 389.67 J/(kg·C°)

*data from lab manual

(i) Calculate and A and ΔA

Use equation: A = πr², r = 2.223cm or 0.02223m, since D = 4.446cm

A = π(0.02223m)² = 0.00155m²

Use Equation: $\dfrac{\Delta A}{A} = \dfrac{\Delta \pi}{\pi} + 2 \cdot \dfrac{\Delta r}{r}$, based on πr². Δr = 0.00001m, since ΔD = 0.002cm

$$\frac{\Delta A}{0.00155m^2} = \frac{0}{3.14} + 2 \cdot \frac{0.00001m}{0.02223m}$$

$$\Delta A = 0.000001395m$$

*data from table 1, 2,

(j) Calculate value of K for rubber

Use equation 5: $K = -\dfrac{mdMC_p}{A}$

$$K = -\frac{(-0.00154)(0.00190m)(0.450kg)(389.67J\,/(kg \cdot C°))}{0.00155m^2} = 0.331J\,/(s \cdot m \cdot C°)$$

*data from table 1, parts (b), (h), (i)

(k) Calculate the error in K for rubber

Use equation: $\dfrac{\Delta K}{K} = \dfrac{\Delta m}{\left| m_{best} \right|} + \dfrac{\Delta d}{d} + \dfrac{\Delta M}{M} + \dfrac{\Delta A}{A} + \dfrac{\Delta C_p}{C_p}$

$$\frac{\Delta K}{0.331} = \frac{0.0001}{\left| -0.00154 \right|} + \frac{0.00002m}{0.00190m} + \frac{0.001kg}{0.450kg} + \frac{0.000001395m^2}{0.00155m^2} + \frac{0}{0}$$

$$\Delta K = 0.0260$$

*data from table 1, parts (b), (f) (h), (i), (j)

(l) Calculate value of K for cork

Use equation 5: $K = -\dfrac{mdMC_p}{A}$

$$K = -\frac{(-0.0001167)(0.00462m)(0.450kg)(389.67J\,/(kg \cdot C°))}{0.00155m^2} = 0.06099J\,/(s \cdot m \cdot C°)$$

*data from table 2, parts (d), (h), (i)

(m) Calculate the error in K for cork

Use equation: $\dfrac{\Delta K}{K} = \dfrac{\Delta m}{|m_{best}|} + \dfrac{\Delta d}{d} + \dfrac{\Delta M}{M} + \dfrac{\Delta A}{A} + \dfrac{\Delta C_p}{C_p}$

$\dfrac{\Delta K}{0.06099} = \dfrac{0.0000143}{|-0.0001167|} + \dfrac{0.00002m}{0.00462m} + \dfrac{0.001kg}{0.450kg} + \dfrac{0.000001395m^2}{0.00155m^2} + \dfrac{0}{0}$

$\Delta K = 0.00793$

*data from table 1, parts (d), (g), (h), (i), (l)

(n) Results

K (rubber) = 0.33 ± 0.03 J / (m · s · C°)

K (cork) = 0.061 ± 0.008 J / (m · s · C°)

*data from part (j), (k), (l), (m)

Discussion

The results of this experiment compared favourably with the expected result of a value on the same order of magnitude as quoted in Cutnell and Johnson [p. 396 (8th edition)]. The below list compares the measured values of K in this experiment with typical values of K for other thermal insulators:

1) Styrofoam – 0.010 J / (s · m · C°)

2) Cork – 0.061 ± 0.008 J / (s · m · C°)

3) Rubber – 0.33 ± 0.03 J / (s · m · C°)

4) Glass – 0.80 J / (s · m · C°)

The K values measured in this experiment compared well against typical K values for rubber and cork. Typically the K value for rubber is 0.2 ± 0.1 J / (s · m · C°) and the K value for cork is 0.02 ± 0.01 J / (s · m · C°). This experiment measured K values which were of the same order of magnitude as the typical values (as listed above), which is considered a good result. While the overlap graphs on the next page do not show an overlap between the typical values and the measured values, the results of the experiment are still considered good as noted above.

There were multiple possible sources of error in this experiment. The copper cylinder used as a heat source was warmed and cooled repeatedly during the experiment. As a result, its temperature may not have stabilized prior to using it as a heat source. As a result, it may have taken more or less time to cool depending on how unstable the cylinder's temperature was. Another source of error was in the time measurements. Since the temperature and time were measured by eye, it was sometimes difficult to determine exactly how long it took for the temperature to drop by a degree. Conversely, when checking the time against the temperature, for the cork test, the temperature reading may have changed by the time the observer realizes that a new temperature measurement should be taken.

Conclusion

The value of thermal conductivity for rubber and cork were calculated successfully in this experiment. The temperature of a copper cylinder was measured with time as it lost heat to an ice-bath container. The slope of the line generated by taking the natural logarithm of the temperatures measured at various times allowed for a calculation of the thermal conductivity, using equation 5. The experiment yielded an acceptable result, which was of the order of magnitude which is comparable to the values quoted in Cutnell and Johnson [p. 396 (8th edition)]. The calculated value for thermal conductivity of rubber is 0.33 ± 0.03 J / (s · m · C°) and the calculated value for thermal conductivity of cork is 0.061 ± 0.008 J / (s · m · C°).

.